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Similarity Theory

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Abstract

Similarity theory provides the theoretical framework for transferring experimental results from laboratory or pilot scale to industrial applications in process engineering. It is based on the principle that systems exhibit similar behavior when their governing dimensionless numbers are identical. This concept enables the prediction of complex process behavior at full scale using controlled experiments at reduced scale.

The article introduces the fundamental concepts of similarity, including geometric, kinematic, and dynamic similarity, and highlights their importance for reliable scale-up. Key dimensionless numbers such as the Reynolds, Prandtl, and Froude numbers are discussed, as they represent the dominant relationships between inertial, viscous, thermal, and gravitational forces in fluid systems.

Special attention is given to the practical limitations of maintaining full similarity in industrial applications. Since not all similarity criteria can typically be satisfied simultaneously, engineering judgment and empirical correlations are required to prioritize the most relevant parameters.

Finally, the application of similarity theory in membrane technology is addressed, where coupled transport phenomena such as flow, mass transfer, and momentum exchange must be considered. The consistent use of dimensionless analysis is essential for designing, optimizing, and scaling membrane processes in a reliable and efficient manner.

Application of Similarity Theory in Membrane Technology

Similarity theory provides a central foundation for transferring experimental results from model scale to industrial applications in process engineering. It makes it possible to systematically describe and quantify the physical relationships between a model system and a full-scale plant. The underlying idea is that two systems are similar if their dimensionless numbers match. This allows experiments to be carried out under controlled conditions at a smaller scale in order to predict the behavior of complex processes at full scale.

In practice, this approach is particularly important in chemical and thermal process engineering. Processes such as distillation, mixing, filtration, and heat transfer are often first studied in pilot plants before being implemented in production plants. Correct scale-up requires that geometric, kinematic, and dynamic similarity be maintained between the model and the real system. These three forms of similarity ensure that the essential physical force relationships are preserved and that no distortions occur in the process parameters.

For the mathematical description, dimensionless numbers such as the Reynolds, Froude, Prandtl, and Nusselt numbers are used. These numbers combine the relevant physical quantities in compact form and make it possible to compare different systems independently of their absolute dimensions. Buckingham's dimensional analysis is a well-established tool for deriving such numbers. It reduces the number of independent variables and allows a systematic investigation of the influencing parameters.

A central objective of similarity theory is to derive reliable predictions for industrial processes from a limited number of model experiments. This can shorten development time, reduce costs, and minimize risks during scale-up. In industrial practice, however, only partial similarity is often achieved, because not all dimensionless numbers can be kept constant at the same time. In such cases, engineering judgment and empirical correlations are essential to find a reasonable balance between theoretical accuracy and practical feasibility.

In addition to its classical hydrodynamic applications, similarity theory is increasingly being used in modern fields such as micro process engineering, environmental engineering, and bioprocess engineering. In these areas, processes must be considered across very different size scales, which makes it even more difficult to maintain similarity conditions. Nevertheless, the basic principle remains unchanged: only by consistently taking dimensionless relationships into account can experimental results be reliably transferred to real systems.

Dimensionless Numbers

The application of similarity theory in process engineering is based on the identification and use of dimensionless numbers. These numbers summarize complex physical relationships in compact form and allow different systems to be compared independently of their geometric dimensions or absolute size. Among the most important are the Reynolds, Prandtl, and Froude numbers. They ensure that the relevant force relationships are preserved when transferring results from a model to the full-scale system.

Reynolds Number

The Reynolds number (Re) is one of the key parameters in fluid mechanics and describes the ratio of inertial to viscous forces in a flow. It is used to distinguish between laminar and turbulent flow and has a major influence on energy input, pressure drop, and mixing in a system. Mathematically, it is defined by the relationship

$$\mathrm{Re} = \frac{\rho\, v\, L}{\mu} \tag{1}$$

where ρ is the density, v is a characteristic velocity, L is a characteristic length, and μ is the dynamic viscosity. In process engineering, the Reynolds number is particularly important for pipe flow, stirred tanks, and fluidized systems. Correct scale-up requires that the Reynolds number be identical in the model and in the full-scale system in order to ensure comparable flow regimes.

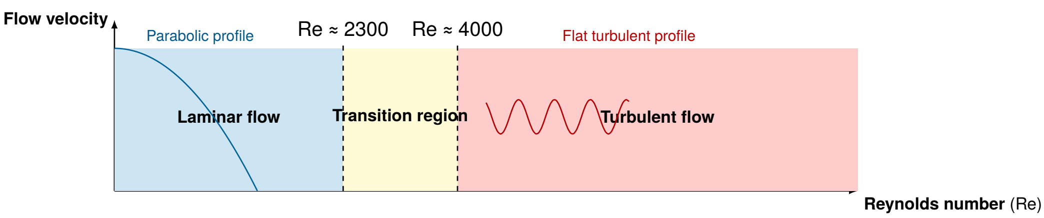

Figure 1. Schematic representation of the Reynolds number and the associated flow regimes. Blue zone: laminar flow (Re < 2300); yellow zone: transition region; red zone: turbulent flow (Re > 4000).

Prandtl Number

The Prandtl number (Pr) relates momentum transport to heat transport in a fluid and plays a central role in heat transfer. It describes the ratio of kinematic viscosity to thermal diffusivity and is defined as follows:

$$\mathrm{Pr} = \frac{v}{\alpha} = \frac{c_p\,\mu}{\lambda} \tag{2}$$

where v is the kinematic viscosity, α is the thermal diffusivity, c_p is the specific heat capacity, μ is the dynamic viscosity, and λ is the thermal conductivity of the fluid. Small Prandtl numbers, such as those of liquid metals, indicate dominant heat transport, while large values, for example in oils, indicate stronger momentum diffusion. The Prandtl number forms the basis of many correlations for heat transfer processes, for example when determining the Nusselt number. For correct scale-up of thermal processes, the Prandtl number must be kept constant in order to ensure similar temperature fields in the model and in the real system.

Froude Number

The Froude number (Fr) characterizes the ratio of inertial forces to gravitational forces in a flow. It is particularly relevant for flows with a free surface, such as those occurring in stirred tanks, overflow processes, or channels. It is defined as:

$$\mathrm{Fr} = \frac{v}{\sqrt{g\,L}} \tag{3}$$

where v is the characteristic velocity, g is gravitational acceleration, and L is a characteristic length. Small Froude numbers indicate gravity-dominated flows, while large values indicate inertia-driven, turbulent conditions. In similarity theory, the Froude number is essential for correctly scaling the dynamics of free surfaces. If it is neglected during scale-up, significant deviations in energy input, flow pattern, or wave formation may occur.

Importance for Scale-Up

In practice, it is rarely possible to preserve all of these dimensionless numbers simultaneously. Engineers therefore have to prioritize which similarity conditions are most important for a given process. For flow processes, the Reynolds number usually dominates; for thermal processes, the Prandtl number; and for free-surface systems, the Froude number. The combination of these parameters forms the basis for physically correct and practically useful scale-up in process engineering.

Number	Meaning	Formula (simplified)
Re	Reynolds number	$\mathrm{Re} = \frac{uL}{\nu}$
Sc	Schmidt number	$\mathrm{Sc} = \frac{\nu}{D}$
Sh	Sherwood number	$\mathrm{Sh} = \frac{k_m L}{D}$
Pe	Péclet number	$\mathrm{Pe} = \mathrm{Re}\,\mathrm{Sc}$
Eu	Euler number	$\mathrm{Eu} = \frac{\Delta p}{\rho u^2}$
Fr	Froude number	$\mathrm{Fr} = \frac{u}{\sqrt{gL}}$

Table 1. Important dimensionless numbers in membrane technology

Application in Membrane Technology

Similarity theory also plays a central role in membrane technology for the description, analysis, and scaling of transport processes. Flow, mass transfer, and momentum transfer are always coupled.

Process variable	Importance for the process
Flow velocity	Determines how fast the fluid flows through the system and influences mass transfer and mixing.
Membrane area	Indicates how much area is available for mass transfer. A larger area allows a higher throughput.
Volumetric flow rate	Controls how much fluid is transported per unit time and how long it remains in the system.
Pressure difference across the membrane	Is the driving force for transport through the membrane.
Temperature	Influences the viscosity of the fluid and the rate of diffusion.
Fluid viscosity	Determines how easily the fluid can flow and how large the flow resistance is.
Solute concentration	Determines how much of a substance is present in the fluid and influences transport to the membrane.
Diffusion coefficient	Describes how quickly particles spread through the fluid.

Table 2. Important process variables in membrane filtration

Flow Velocity

Flow velocity describes how fast the fluid moves along the membrane surface. It directly determines the hydrodynamic regime and is therefore central to the Reynolds number (Re = $\frac{uL}{\nu}$). Higher velocities increase Re, promoting turbulent or transitional flow, which enhances mixing and reduces concentration polarization. Through its influence on mass transfer, flow velocity also affects the Sherwood number (Sh) and the Péclet number (Pe = Re Sc). Thus, it is a key parameter for controlling transport phenomena near the membrane.

Membrane Area

The membrane area defines the available surface for mass transfer and determines system throughput. While it does not directly appear in classical dimensionless numbers, it influences characteristic length scales (L) used in Re, Sh, and Pe. A larger membrane area allows operation at lower local fluxes, which can reduce concentration polarization and fouling, indirectly affecting mass transfer correlations expressed through the Sherwood number.

Volumetric Flow Rate

The volumetric flow rate determines how much fluid is processed per unit time and is directly linked to flow velocity. Through this relationship, it influences the Reynolds number and therefore the flow regime. Changes in volumetric flow rate affect shear conditions at the membrane surface, which in turn impact the Sherwood number and mass transfer coefficients. As a result, it also affects the Péclet number, representing the balance between convective and diffusive transport.

Pressure Difference Across the Membrane

The pressure difference across the membrane is the driving force for permeation. It primarily affects convective transport through the membrane rather than the external flow field. However, increased permeation flux can intensify concentration polarization, thereby altering local concentration gradients and influencing the effective Sherwood number. In systems with strong convective transport, pressure-driven flux also contributes to the Péclet number by increasing the relative importance of convection over diffusion.

Temperature

Temperature affects fluid properties such as viscosity and diffusion coefficients. A decrease in viscosity with increasing temperature leads to higher Reynolds numbers, while an increase in diffusion coefficients reduces the Schmidt number (Sc = $\frac{\nu}{D}$). These combined effects influence the Sherwood number and overall mass transfer. Temperature is therefore a key parameter linking hydrodynamics and transport phenomena through multiple dimensionless numbers.

Fluid Viscosity

Viscosity represents the resistance of the fluid to flow and is a central parameter in the Reynolds number. Higher viscosity lowers Re, promoting laminar flow conditions and reducing mixing. It also increases the Schmidt number, as Sc = $\frac{\nu}{D}$, which indicates slower diffusive transport relative to momentum transport. As a result, viscosity strongly affects both the Sherwood and Péclet numbers and plays a critical role in determining mass transfer efficiency.

Solute Concentration

Solute concentration influences transport processes through its effect on concentration gradients and osmotic pressure. While it does not directly appear in classical dimensionless numbers, it affects the driving force for diffusion and thus the effective mass transfer coefficient. High concentrations can enhance concentration polarization, altering the boundary layer behavior and thereby influencing the Sherwood number. In convective-diffusive systems, it also affects the interpretation of the Péclet number.

Diffusion Coefficient

The diffusion coefficient is a key parameter for mass transport and directly appears in the Schmidt number (Sc = $\frac{\nu}{D}$) and the Sherwood number (Sh = $\frac{k_m L}{D}$). A higher diffusion coefficient leads to lower Sc and improved mass transfer, resulting in higher Sherwood numbers. It also influences the Péclet number, which describes the ratio of convective to diffusive transport. Therefore, the diffusion coefficient is fundamental for quantifying transport efficiency in membrane systems.

Symbols

Symbol	Meaning	Unit (SI)
Δp	Pressure difference	Pa
ΔT	Temperature difference	K
$\dot{m}$	Mass flow rate	$kg\,s^{-1}$
$\dot{Q}$	Volumetric flow rate	$m^3\,s^{-1}$
γ	Shear rate	s^{-1}
ρ	Density	$kg\,m^{-3}$
η	Dynamic viscosity	Pa s
ν	Kinematic viscosity	$m^2\,s^{-1}$
p	Pressure	Pa
T	Temperature	K
u	Average flow velocity	$m\,s^{-1}$
V	Volume	m^3
t	Time	s
g	Gravitational acceleration	$m\,s^{-2}$

Table 3. Important quantities and their units